Semiclassical Aspects of Quantum Mechanics by Classical Fluctuations

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Abstract

Building on a model recently proposed by F. Calogero, we postulate the existence of a universal Keplerian tremor for any stable classical system. Deriving the characteristic unit of action α for each classical interaction, we obtain in all cases $\alpha \cong h$, the Planck action constant, suggesting that quantum corrections to classical dynamics can be simulated through a fluctuative hypothesis of purely classical origin.

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In a recent paper F. Calogero has put forward an intriguing conjecture on the possible gravitational origin of quantization [1].

The scheme followed by Calogero is simple but appealing. He suggests that the origin of quantization be attributed to the universal interaction of every particle with the background gravitational force due to all other particles

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in the Universe. Such background interaction generates a chaotic component in the motion of each single particle, with a characteristic constant τ measuring the time scale of stochasticity (Zitterbewegung).

Assuming a basic granularity of the Universe, made up of nucleons (or hydrogen atoms) of mass m, Calogero derives an expression for Planck's constant, $h \cong G^{1/2} m^{3/2} R^{1/2}$, with G the Newtonian gravitational constant and R the observed Radius of the Universe. This formula, which connects the fundamental constant of quantum theory with the fundamental gravitational constants, was already known since some time [2] but its meaning and implications were so far unexplained; Calogero provided the first derivation of it from a mechanical model, so that one could very well name it the Calogero–Weinberg formula.

Now, the crucial point in the procedure carried out by Calogero is that the characteristic time τ of the stochastic motion per particle, being associated to a collective chaotic effect, should be inversely proportional to the square root of N, the total number of particles in the Universe [1]:

$$\tau \cong N^{-1/2} \mathcal{T} \,, \tag{1}$$

with $\mathcal{T}$ the characteristic global time unit associated with a Universe of total mass M. Defining the energy per particle $\epsilon \cong E/N$, with E total energy of the Universe, and a global unit of action for the Universe $A = E\mathcal{T}$, Calogero defines the unit of action per particle

$$\alpha = \epsilon\tau \cong N^{-3/2} A \,. \tag{2}$$

Replacing N with the ratio of the global and the granular amount of sources M/m, imposing that α be independent of extensive quantities and performing an elementary dimensional analysis for the combination of the nucleon mass m, the Radius of the Universe R and the Newtonian gravitational constant G, Calogero finally arrives at the expression

$$\alpha \cong G^{1/2} m^{3/2} R^{1/2} \,; \tag{3}$$

inserting the numerical values $m \cong 10^{-27} kg$, $G \cong 10^{-11} kg^{-1} \cdot m^3 \cdot s^{-2}$ and the most updated cosmological estimate for the observed Radius of the Universe $R \cong 10^{30} m$ [3], eq.(3) yields $\alpha \cong h$, the Planck action constant (we warn the reader that in the presente work we are neglecting in the numerical

computations all those factors that do not substantially affect the order of magnitude of the estimated quantities).

In conclusion, Calogero suggests that quantization might be explained via classical gravitation by assuming the existence of a chaotic component of the individual particles' motion due to a "universal coherent tremor" associated to the extremely large number N of the elementary components making up the Universe. This conjecture is mathematically implemented introducing Maxwell–Boltzmann fluctuations proportional to $1/\sqrt{N}$.

In the present letter we take a closer look at the model provided by Calogero. We first show that his procedure can be equivalently reformulated by replacing the fluctuative law eq.(1) with a fractal space–time relation $l \sim \tau^{2/3}$ which is the mathematical expression of a "universal Keplerian tremor".

We then move on to show that the scheme of Calogero can be applied to all the other known interactions (electromagnetic, strong, etc.) and that it leads in all cases to a variety of formulas again linking Planck's constant with the proper fundamental constants associated to each considered interaction.

In this way we achieve two purposes: on the one hand we clarify that the fluctuative hypothesis of Calogero actually holds also for systems with few degrees of freedom, since in our Keplerian reformulation it does not involve the number N of elementary constituents. On the other hand, we show that the mechanism is universal, in the sense that it allows to derive an expression for Planck's constant for any physical system confined on the typical space–time region associated to any of the fundamental interactions known in Nature.

In particular, one can apply the Keplerian formulation of the Calogero model to "Gedanken Universes" made of an arbitrary number N of gravitationally interacting particles, even with $N = 2$. In any instance, one always obtains $\hbar$ as the unit of action and the observed Radius of our actual Universe as the typical length scale of the system, irrespective of the assumed number N of elementary constituents. Therefore the Keplerian fluctuation and its relationship with the elementary quantum of action $\hbar$ seem to have little or nothing at all to do with a chaotic dynamics induced by the enormous number N of elementary constituents that interact gravitationally in the actually observed Universe.

We can then draw the following conclusions:

1) The mechanism envisaged by Calogero seems to capture some essential

aspects of the interplay between classical and quantum mechanics.

2) The universality of the mechanism as shown in the present letter and its insensitivity to N being large or small strongly suggest that the simple and appealing interpretation of Calogero is untenable: quantum mechanics is in fact the most fundamental theory known up to now, and the derivation of Planck's constant for each known interaction by this procedure rules out the possibility of a privileged role of classical gravitation as the "origin" of quantum mechanics.

3) Still it is remarkable that a simple qualitative argument (the universal Keplerian tremor) allows to capture some relevant quantum features by purely classical considerations.

4) From the point of view of quantum mechanics we can then interpret Calogero's and our results as a first embryonal, still qualitative, step towards the possibility of recovering semiclassical aspects of quantum mechanics starting directly from classical mechanics and implementing (simulating) quantum corrections in terms of suitable, purely classical, stochastic fluctuations. This is at variance with the usual approximation schemes of quantum mechanics, such as the WKBJ procedure, which recover the semiclassical and classical domains starting from the "deep" quantum domain. It is rather a first operative definition of a semi–quantal approximation scheme, hopefully to be further developed.

We begin by showing that the tremor hypothesis of Calogero, eq. (1) is equivalent to a generalization on the microscopic scale of Kepler third law in the form $l \sim \tau^{2/3}$, where, by introducing the total volume of the Universe V and the mean allowed volume per particle (specific volume) $v_s \cong V/N$, we have defined the mean free path of the individual constituents $l \cong v_s^{1/3}$. In fact, we can immediately rewrite Calogero's fundamental relation eq. (1) as:

$$\frac{\mathcal{T}^2}{V} \equiv const. \cong \frac{\tau^2}{v_s} \sim \frac{\tau^2}{l^3} \, , \tag{4}$$

or, equivalently,

$$l \sim \tau^{2/3} \, . \tag{5}$$

Note that, from $V \sim R^3$, the first member of eq. (4) is Kepler third law on the scale of the Universe.

One should note that the most updated cosmological scenarios [3] lead to a recessing away law of galaxies in the expanding Universe of the form

$L \sim t^{2/3}$ (with L the distance between galaxies). The congruence of this phenomenon on large cosmological scales with our Keplerian version eq. (5) of Calogero's tremor hypothesis eq. (1) implies the extension of validity of a Kepler–like third law for gravitational interactions ranging from small to large scales. The resulting picture clearly ignores the structure of the system in its finest details, being based on a sort of mean field description.

At this point we might draw the first provisional conclusion: if one assumes, as usual, that quantum mechanics is the fundamental theory, then the above analysis implies that the stability of the Universe on the scale of its observed Radius R is ruled, via the Calogero mechanism as formulated in eq. (5), by the Planck quantum of action, in complete analogy with the stable confined systems associated to the other known interactions.

This interpretation is of course totally at variance with the one originally given by Calogero, and to see whether it is tenable, we should move on to apply his model to the other stable systems on different scales that are associated to the other relevant known interactions beyond gravity.

We expect in this way to obtain again formulas linking Planck's action constant to the typical radius R of stability and to the fundamental masses and interaction constants associated to the systems being considered.

As we will show below, it turns out that this is indeed the case. This fact seems to imply that a simple qualitative picture based on purely classical tools allows to recover some fundamental aspects of quantum mechanics in its semiclassical domain in connection with the stability of matter.

We thus proceed to apply the scheme already adopted for gravitation to aggregates of charged particles interacting electromagnetically and to systems of confined quarks in the nucleons. We do this by assuming the tremor hypothesis in the form of eq.(5) to hold for any stable aggregate of particles, relying on the fact that for these confined systems one can certainly introduce well defined characteristic global units of time $\mathcal{T}$ and volume V.

Electromagnetic Interactions. Let us consider first the case of a stable aggregate of charged particles interacting via electromagnetic forces. The fundamental constants involved are the electrostatic constant $K \equiv 1/4\pi\epsilon_0 \cong 10^{10} N \cdot m^2 \cdot C^{-2}$, the elementary unit of charge $e \cong 10^{-19} C$ and the velocity of light $c \cong 10^8 m \cdot s^{-1}$.

Since such aggregates are in general made of collections of electrons and protons, we can take as the natural unit of elementary mass the reduced

mass μ, which substantially coincides with the mass of the electron $m \cong 10^{-30}kg$. Let us consider also the characteristic linear dimension R of the stable aggregate; this characteristic global scale of length can vary from $R \cong 10^{-2}m$ (macroscopic dimensions) to $R \cong 10^{-10}m$ (atomic dimensions).

Expressing N as the ratio of the global and the granular amount of sources Q/e, with Q total charge of the aggregate, imposing eq. (2) and requiring the independence of the unit of action α on extensive quantities, we obtain $A = Q^{3/2}\tilde{A}$; by dimensional considerations $\tilde{A} = f(K, m, c, R)$, and we finally arrive at

$$\alpha \cong e^{3/2}K^{3/4}m^{1/4}c^{-1/2}R^{1/4} \, . \qquad (6)$$

Inserting numbers in eq. (6) we have then in all cases, up to at most one order of magnitude, $\alpha \cong 10^{-34}J \cdot s \cong h$, i.e., once more, Planck constant.

Quarks. We now move on to consider a hadron having as granular costituents a collection of bound quarks. The interaction we consider is the "string law" described by the typical confining potential $V = kr$ with the strength constant k varying in the range $k \cong 0.1GeV \cdot fm^{-1} \div 10GeV \cdot fm^{-1}$ (values compatible with the experimental bounds [4]). Let us also introduce the quark masses $m \cong 0.01GeV \cdot c^{-2} \div 10GeV \cdot c^{-2}$ [6], the velocity of light c and the radius $R \cong 10^{-15}m$, which is the range of nuclear forces.

Expressing N as $N = M/m$, M total mass of the hadron, we obtain, following the usual procedure, $A = M^{3/2}\tilde{A}$ and, finally,

$$\alpha \cong (mc^2)^{3/2}c^{-1}k^{-1/2}R^{1/2} \, . \qquad (7)$$

Inserting numbers, we have again, up to at most one or two orders of magnitude, $\alpha \cong h$.

This numerical equivalence with Planck constant of the elementary unit of action per particle for any classical fundamental interaction on each scale seems very significant, and can hardly be thought of being casual. We again stress that one always obtains the same order of magnitude of α ($\cong h$) for any force law on its typical scale (universality of Planck constant).

Since the above procedure is of a grossly qualitative nature, it seems important to provide other consistency checks of the fundamental Keplerian tremor law eq. (5), to yield further support to its universal validity.

In fact, the natural question arises on how to determine the order of magnitude, for each particular system, of the characteristic time τ. The

latter must obviously depend both on the universal elementary unit of action $\alpha \cong h$ and on the details of the chosen aggregate.

It is well known that the typical time scale of quantum fluctuations is defined as the ratio between h and a suitable energy describing the equilibrium state of the given system on its characteristic dimensions. This leads naturally to identify this energy with the thermal energy $k_B T$, with k_B the Boltzmann constant and T the absolute temperature.

On the other hand, in our model such time scale coincides with the fluctuative time τ; we therefore write

$$\tau \cong \frac{h}{k_B T} \, . \tag{8}$$

With the above definition we can rewrite the universal tremor hypothesis eq. (1) in the form:

$$T \cong \frac{h}{k_B} \cdot \frac{\sqrt{N}}{\mathcal{T}} \, . \tag{9}$$

We can adopt the point of view that the above equation be the definition of the absolute temperature for the system one is considering. This definition, as one can see, connects the temperature to the global and the granular length scales, and to the characteristic velocity associated to the given aggregate.

We now exploit this definition, applying it to some well established thermodynamic phenomena, as a further test of validity of our theoretical scheme.

a) Emittance associated to charged beams in particle accelerators. Among the stable aggregates of charged particles, a paradigmatic role is played by the charged beams in particle accelerators. Such systems are very interesting from our point of view because they are generally described in classical terms, since they exist on a length scale that ranges approximately from thirtytwo to thirtyfive orders of magnitude above the Planck scale.

However, our analysis shows that their stability, as in the gravitational case, is ruled by Planck's constant, and it is thus ultimately of quantum rather than classical origin.

The bunch consists solely of charges of the same sign, and stability (confinement) can be achieved only through the action of an external focusing potential (magnetic field). Our analysis applies, for instance, by considering the reference frame comoving with the synchronous particle, yielding again h as the unit of action per particle (note that replacing electrons with protons

does not affect in a appreciable way the order of magnitude of α due to the $m^{1/4}$ dependence in eq. (6)).

The emittance $\mathcal{E}$ is a scale of length (or ,equivalently, of “temperature”) associated to charged beams, whose numerical value in units of the Compton length $\lambda_c = h/mc$ lies, for typical accelerators (for instance electron machines), in the range $\mathcal{E} \cong 10^6 \lambda_c \div 10^9 \lambda_c$ [5]. Following our scheme, we identify the characteristic unit of emittance as the characteristic action associated to the charged beam divided by mc. The characteristic action associated to charged beams is, in our framework, $(k_B T)\mathcal{T}$, and therefore, by eq. (9), the associated emittance is estimated as:

$$\mathcal{E} \cong \lambda_c \sqrt{N} \, . \tag{10}$$

We note that the above expression connects, at least in the leading semiclassical order, the characteristic emittance with the number of particles in a nontrivial way. Since in a typical bunch $N \cong 10^{11} \div 10^{12}$ [6], we finally obtain $\mathcal{E} \cong 10^6 \lambda_c \cong 10^{-6} m$ in good agreement with the phenomenological order of magnitude estimated by other theoretical methods [5].

Therefore the Calogero model supplemented by eq.(8) naturally supports a quantum–like description of the dynamis of charged particle beams with the correct order of magnitude for the emittance.

b) Temperature of macroscopic systems. In this case we know that $N \cong 10^{23} mol^{-1} \div 10^{24} mol^{-1}$ (Avogadro constant), and $\mathcal{T} \cong 10^{-2} s \div 10^{-3} s$, corresponding to a rms velocity $v_T \cong 10^2 m \cdot s^{-1} \div 10^3 m \cdot s^{-1}$ for gases around room temperature. Inserting the numerical values of h/k_B we obtain $T \cong 10^2 K \div 10^3 K$, as it should be.

c) Temperature of quarks inside nucleons. In this case $N \cong 1$. The typical energy scale E of light quarks in a nucleon is of the order of $\Lambda_{QCD} \cong 0.1 GeV$ [6], corresponding to a temperature $T = E/k_B \cong 10^{12} K$. With the characteristic velocity of the order of the velocity of light c, and the global scale of length $R \cong 10^{-15} m$, we have $\mathcal{T} \cong R/c \cong 10^{-23} s$. Inserting numbers into eq. (9) we obtain just $T \cong 10^{12} K$.

d) Bose–Einstein condensation. Recently, Bose–Einstein condensation has been experimentally observed in a gas of rubidium and sodium atoms [7]. The condensate has linear dimension $R \cong 10^{-4} m$ at a temperature $T \cong 10^{-6} K$ and it contains $N \cong 10^7$ atoms. Letting $\mathcal{T} \cong R/v$, with v the

characteristic velocity, eq. (9) yields

$$v \cong \frac{k_B}{h} \cdot \frac{T}{\sqrt{N}} \cdot R \cong 10^{-3} m \cdot s^{-1} \div 10^{-2} m \cdot s^{-1} \,. \tag{11}$$

The characteristic velocity thus is smaller by a factor of the order 10^{-6} compared to the rms velocity of the gas observed at room temperature, in agreement with the theoretical prediction of a macroscopic "zero" momentum.

Therefore, the definition of temperature derived from the universal Keplerian tremor hypothesis seems to be consistent. We then move on to apply it to other two significant cases.

e) Cosmic background radiation. In the framework of our hypothesis it seems quite reasonable to interpret the measured temperature associated to the cosmic background radiation, $T = 2.7K$, as the characteristic "temperature of the Universe". Consequently, we insert in eq. (9) a characteristic global time $\mathcal{T} \cong R/v$, with R the Radius of the Universe and v a characteristic velocity. This velocity cannot be defined unambigously, therefore we take it in the wide range that goes from $10^5 m \cdot s^{-1}$ (the circular velocity of hydrogen clouds surrounding galaxies) up to the velocity of light.

We now exploit eq. (9) to determine the order of magnitude of N, the total number of particles in the Universe. Inserting numbers:

$$N \cong 10^{66} \div 10^{72} \,, \tag{12}$$

which, in our crudely qualitative framework, is compatible, within the error range, with the value $N \cong 10^{\nu}$, $\nu = 78 \pm 8$, estimated by cosmological arguments [3].

Some conclusive remarks are now due. We first want to stress that, according to our point of view, it is impossible to discriminate the gravitational system, through the observed Radius of the Universe R, from the other systems (e.g. charged particles and quarks) by claiming that its characteristic dimensions are not *a priori* determined by quantum mechanics while the dimensions of the other ones are.

In fact, one should note that there are about 20 orders of magnitude separating the smallest scale of length considered (that of the quarks confined in the nucleons) and the Planck scale representing the fundamental quantum

mechanical scale of length. Furthermore, if one considers **macroscopic** aggregates like charged beams in particle accelerators, this difference reaches up to 35 orders of magnitude.

It seems then obvious to us that if one accepts, as it is commonly asserted, that the influence of quantum mechanics should manifest itself all the way through this huge difference of length scales, it should as well manifest itself also on the scale of length of the Universe.

Therefore, the significant aspect that we single out in the scheme put forward by Calogero is that it allows for any dynamical system to obtain a quantum correction to classical dynamics starting from a fluctuative hypothesis of purely classical origin. This model seems then worth to be developed and improved, since it could be of great conceptual and computational relevance in the study of the interplay between classical and quantum domains, a fundamental issue in modern physics. We will report in a forthcoming paper how the application of these ideas can be made already now quantitative in the study of the quantum–like dynamics of charged particle beams in accelerators.

¿From both the conceptual and the calculational point of view the next task would then be to develop the present qualitative model into a fully quantitative scheme of approximation (semi–quantal approximation scheme) that would allow, in principle, to determine all the higher–order quantum corrections, i.e. a systematic reconstruction of quantum fetures from classical dynamics beyond the leading semiclassical order.

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